Problem 5: UConn men's basketball tourney chances

Starting on March 12, the UConn men's basketball team will play in the AAC Championship (at the XL Center in Hartford) for up to 3 single-elimination games. If UConn wins the 1st game, then it will play the 2nd game, and if it wins again, then it will play the 3rd and final game. However, a single loss will eliminate UConn from the AAC Championship.

Suppose UConn wins the 1st game with probability 0.6. Given that it wins the 1st game, the probability of winning the 2nd game is 0.5. And given that it wins the first 2 games, the probability of winning the 3rd and final game is 0.4.

Based on the outcome of the AAC Championship, a selection committee decides whether UConn advances to the NCAA tournament. By rule, if UConn wins all 3 games, then it advances to the tournament with probability 1 ("automatic bid"). If UConn wins 2 games (and loses the 3rd), the probability of advancing is 0.2. If UConn wins fewer than 2 games, there is 0 probability of advancing.

(a) Draw a tree diagram, and find the probability that UConn advances to the NCAA tournament under the above scheme.

..... (more on the next page)

(b) Given that UConn advances to the NCAA tournament, what is the probability that it did not win all 3 games in the AAC Championship? **SIMPLIFY** your numerical answer.

..... END EXAM

Useful formulas:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \qquad \binom{n}{n_1, \cdots, n_r} = \frac{n!}{n_1! \cdots n_r!} \quad (\text{if } n_1 + \cdots + n_r = n)$$
$$(x+y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k} \qquad (x, y \in \mathbb{R}, n \in \mathbb{N})$$

(Bayes) If $\bigcup_{j=1}^{N} F_j = S$ and the F_j are mutually disjoint, then $P(F_i|E) = \frac{P(E|F_i)P(F_i)}{\sum_{j=1}^{N} P(E|F_j)P(F_j)}$.