UCONN
UNIVERSITY OF CONNECTICUT

MATH 3160 - Probability - Fall 2014

Midterm Exam 2 (November 5, 2014)

Name: _____

Section (circle one): 004 005

#	Topic	Score
1	(Yik Yak I)	
2	(Yik Yak II)	
3	(Yik Yak III)	

#	Topic	Score
4	(Yik Yak IV)	
5	(Yik Yak V)	
Total		

Instructions:

- Each of the 5 problems in this exam is worth 20 points. The total score for this exam is 100 points.
- **All necessary work must be shown** in the space provided. Answers consisting of purely numbers, without any explanation (via words or diagrams) or derivation, will receive no credit, even if the answers turn out to be correct. Unintelligible answers may also receive no credit. For your benefit, cross out any scratch work which you do not wish graded.
- **Make sure that your answer is logically sound and notationally precise. Do not write anything that is technically untrue.** Points will be docked for stating mathematical nonsense.
- **If you introduce symbols which are not declared in the problem, make sure to explain their meanings as they relate to the problem.** *Example:* "Let X (a random variable) represent the number of typos in a book page. We model X according to the (blah) distribution." Points will be docked if your answer contains obscure, confusing, or conflicting symbols.
- **Please give numeric answers** according to the instructions for each problem. It is okay to leave your answer in terms of exponentials, factorials, or square roots.
- The table of probability distributions and the Z-table are provided on the **last 2 pages**. You may tear off those pages as you wish.
- Only your writing utensils and two 8x11 pages of handwritten notes are allowed during this exam. **No book, calculator, or electronic device may be used.** Any violation of these rules will automatically result in a zero on this exam.
- You have 50 **minutes** (+5 minutes which bleed into the break) to complete this exam. **Good luck!**

EXAM BEGINS ON PAGE 2

Problem 1: Yik Yak I

Yik Yak is a popular location-based microblogging service which has taken hold of the Storrs-Mansfield area. Unlike Facebook or Twitter, Yik Yak users remain anonymous by default. Given a randomly chosen Yik Yak college user in Mansfield aged 18 to 21, let

$$X = \begin{cases} 1, & \text{if the user goes to UConn,} \\ 0, & \text{if the user goes to Eastern CT State Univ.,} \end{cases}$$

and Y be the user's age. Suppose the joint distribution of X and Y is given by the following joint probability table:

X \ Y	18	19	20	21
0	0.1	p	0.05	0.05
1	$2p$	0.2	0.2	p

(a) Find p.

(b) Find the marginal distribution of X and of Y, respectively. Are X and Y independent? Justify briefly.

Problem 2: Yik Yak II

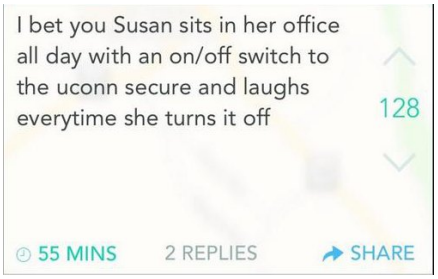
You are monitoring Yik Yak posts (to be referred to as "Yaks" from now on) arriving on your smartphone. Suppose they arrive according to a Poisson process with a rate of 10 Yaks per minute. (**PLEASE** simplify your answers as much as possible.)

(a) Over the course of 3 minutes, what are the mean and the standard deviation, respectively, of the number of Yaks you will have seen?

(b) Suppose at 11 hours, 30 minutes, 10 seconds[1] you saw a Yak appear on your smartphone. What is the probability that the next Yak appears after 11 hours, 30 minutes, 30 seconds?

[1] which is about the time you got to this problem.

Problem 3: Yik Yak III



Consider the Yak shown on the right. Assume that anyone who views this Yak would independently either vote it up ($\wedge$) with probability $4/5$, or vote it down ($\vee$) with probability $1/5$. The "Yakarma score" of a given Yak is defined to be the number of "up" votes minus the number of "down" votes.

If 400 viewers voted on this Yak:

(a) Find the mean and the standard deviation of its "Yakarma score." Simplify your answer.
[*Hint:* $4 \times 8 = 32$, $5 \times 8 = 40$.]

(b) What is the probability that there are more than 336 "up" votes (and fewer than 64 "down" votes, resulting in a "Yakarma score" of more than 272)? Please give a numerical answer to 4 decimal places.
[*Hint:* Use the central limit theorem.]

Problem 4: Yik Yak IV

Suppose the developers of Yik Yak decide to monetize their contents by placing advertisements alongside each Yak. Let X be the lifetime (in minutes) of a given Yak, which is modeled by an exponential random variable of parameter 2. Let $Y = e^{2X}$ be the potential ad dollars (say, in bitcoins) earned from the Yak.

(a) Find the cdf $\mathbb{P}[Y \leq y]$ of Y. Justify all derivations.

(b) Find the pdf $f_Y(y)$ of Y by differentiating the corresponding cdf you obtained in Part (a). Be careful to state the support of the pdf.

Problem 5: Yik Yak V

Let (X, Y) denote the Cartesian coordinate of a random Yik Yak user within a 1-mile radius of E.O. Smith High School. Suppose the associated probability density function is given by[2]

$$f_{X,Y}(x,y) = \begin{cases} C\sqrt{x^2+y^2}, & \text{if } \frac{1}{4} \le (x^2+y^2) \le 1, \\ 0, & \text{if } (x^2+y^2) < \frac{1}{4} \text{ or } (x^2+y^2) > 1. \end{cases} \tag{1}$$

(a) Find the constant C. [*Hint:* Express the double integral using polar coordinates.]

(b) Let $D = \sqrt{X^2+Y^2}$. Find $\mathbb{E}[D]$, the mean distance of a Yik Yak user from the center of E.O. Smith. (Simplify your answer as much as possible.)

..... END EXAM .....

[2]The zero density within a half-mile radius takes into account the reality that many high schools have banned Yik Yak from their campuses. However this author does not know with probability 1 whether Yik Yak is banned at E.O. Smith.