MATH 3170 - Elementary Stochastic Processes - Spring 2014

Course schedule

(Final)

Unless otherwise noted, all readings refer to *Essentials of Stochastic Processes* (2nd ed.) by Durrett. Either the on-line beta version or the printed final version is fine. It is strongly recommended that you read the assigned sections before the indicated lecture.

There will be two midterm exams and a final exam, all of which are take-home. I will post each exam on Piazza on the dates indicated below, and ask you to submit the completed exam to my office by the following Thursday. The midterm exam (resp. final exam) format will be like a 75-minute-long (resp. 2-hour-long) in-class exam, but you will have 4 days (resp. 7 days) to complete it at home, with open book and notes. During exam weeks there will be no homework assignments due.

Dr. Thomas Laetsch will guest lecture on Tu 3/25 and Th 3/27.

Contents in magenta are clickable URL links.

Wk	Date	Topic(s)	Readings	HWs
1	Tu 1/21	Class intro. Discrete-time Markov chains: set-up and examples.	1.1	
	Th 1/23	More examples of Markov chains. (Begin) multistep transition probabilities.	1.2	
2	Tu 1/28	The Chapman-Kolmogorov equations. Recurrence vs. transience.	1.3 (up to Lem 1.6)	
	Th 1/30	Stopping times and the strong Markov property. How to tell if a state is recurrent or transient.	Rest of 1.3	**HW1 Due**
3	Tu 2/4	Expected number of visits. Irreducibility. The decomposition theorem for finite state space Markov chains. Stationary distributions.	1.4	
	Th 2/6	Social networks and (un)directed graphs [in honor of the 10th anniversary of Facebook]. The **Q** matrix associated with a Markov chain. Doubly stochastic chains. Periodicity.	1.5∼1.6.1	
4	Tu 2/11	(Finish) periodicity. Stating the limit theorems without proof. Detailed balance.	1.6.2∼1.6.3	**HW2 Due**
	Th 2/13	Reversible chains and examples. (Begin) one-step calculations. *(On-line lecture due to snow day.)*	1.8∼1.9	
5	Tu 2/18	(Finish) one-step calculations. A sketch of the coupling argument used to prove the ergodic theorem.	1.7	
	Th 2/20	Homework matters. Random walk on $\mathbb{Z}^d$ and Pólya's theorem.	See slides	**HW3 Due**
6	Su 2/23	**Midterm exam 1 out**	**1.1∼1.9**	
	Tu 2/25	Proofs of the limit theorems. Markov chains on infinite state space. Reflected random walk on the line.	1.10, Ex. 1.51	
	Th 2/27	Null recurrence. Galton-Watson branching processes.	Ex. 1.52 & 1.53	
		Midterm exam 1 due by 6pm		
7	Tu 3/4	Finish Galton-Watson process. Quick review of Poisson & exponential random variables. Intro to homogeneous Poisson processes.	2.1∼2.2	

	Th 3/6	Properties of homogeneous Poisson processes.	2.2	**HW4 Due**
8	Tu 3/11	Compound Poisson processes. Random sums.	2.3	
	Th 3/13	Thinning, superposition, and conditioning.	2.4	
9		*Spring break*		
10		**Guest lecture all week: be nice to the lecturer!**		
	Tu 3/25	Renewal processes.	3.1	
	Th 3/27	Age and residual life	3.3	**HW5 Due**
11	Tu 4/1	Continuous-time Markov chains: definition, and construction from discrete-time chains.	4.1	
	Th 4/3	How to compute transition probabilities. The (backward & forward) Kolmogorov equation ($\frac{d}{dt}p_t = \mathbf{Q}p_t = p_t\mathbf{Q}$).	4.2	**HW6 Due**
12	Su 4/6	**Midterm exam 2 out**	**1.10∼3.3**	
	Tu 4/8	Stationary distributions & limit behavior. A baby queueing example.	4.3	
	Th 4/10	Markovian queues. A quick primer on conditional expectation.	4.5, 5.1	
	F 4/11	**Midterm exam 2 due by 6pm**		
13	Tu 4/15	Martingales: definition and examples.	5.2	
	Th 4/17	The optional stopping theorem. Brownian motion.	See notes	**HW7 Due**
14	Tu 4/22	Intro to mathematical finance. The one-step binomial options pricing model.	6.1∼6.2	
	Th 4/24	The one-step binomial options pricing model (cont.)	See notes	**HW8 Due**
15	Tu 4/29	The fundamental theorem of finance (existence of a risk-neutral measure). Multi-step binomial options pricing model.	See notes	
	Th 5/1	The continuous-time Black-Scholes model and formula. The grand finale.	See notes	**HW9 Due**
		Final exam out		
16	Th 5/8	**Final exam due by 12:30pm**		

Exam coverage

- The 1st midterm exam covers exclusively discrete-time Markov chains on a finite state space.
- The 2nd midterm exam covers discrete-time Markov chains on an infinite state space, Poisson processes, and renewal theory.
- The final exam is cumulative, but with a strong emphasis on the last third of the course: continuous-time Markov chains, martingales, Brownian motion, and mathematical finance.